Journal of Biomechanics 48 (2015) 907–910

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Examining the response pressure along a fluid-filled elastic tube to comprehend Frank's arterial resonance model

Yuh-Ying Lin Wang [a,*], Wah-Keung Sze [a], Chin-Chih Lin [a], Jiang-Ming Chen [a], Chin-Chi Houng [a], Chi-Wei Chang [c], Wei-Kung Wang [b,c]

[a] Department of Physics, National Taiwan Normal University, Taipei 116, Taiwan
[b] Institute of Physics, Academia Sinica, Taipei 115, Taiwan
[c] Graduate Institute of Biomedical Electronics and Bioinformatics, National Taiwan University, Taipei 106, Taiwan

ARTICLE INFO

Article history:
Accepted 15 February 2015

Keywords:
Arterial resonance
Wave equation
Pressure wave

ABSTRACT

Frank first proposed the arterial resonance in 1899. Arteries are blood-filled elastic vessels, but resonance phenomena for a fluid-filled elastic tube has not drawn much attention yet. In this study, we measured the pressure along long elastic tubes in response to either a single impulsive water ejection or a periodic water input. The experimental results showed the low damped pressure oscillation initiated by a single impulsive water input; and the natural frequencies of the tube, identified by the peaks of the response in the frequency domain, were inversely proportional to the length of the tube. We found that the response to the periodic input reached a steady distributed oscillation with the same period of the input after a short transient time; and the optimal pressure response, or resonance, occurred when the pumping frequency was near the fundamental natural frequency of the system. We pointed out that the distributed forced oscillation could also be a suitable approach to analyze the arterial pressure wave. Unlike Frank's resonance model in which the whole arterial system was lumped together to a simple 0-D oscillator and got only one natural frequency, a tube has more than one natural frequency because the pressure $P(z,t)$ is a 1-D oscillatory function of the axial position z and the time t. The benefit of having more than one natural frequency was then discussed.

1. Introduction

Natural frequencies can represent the coupling properties of a system. Arterial resonance was first proposed and studied quantitatively by Otto Frank (1899). Remington and Hamilton (1945, 1947) initiated the "natural frequency" of the blood vessels. Most researchers of hemodynamic took the axial blood motion as the major momentum equation in arteries (Milnor, 1989; Li, 2000; Parker, 2009) and due to the high dissipation in the blood flow, those resonance models were neglected after the challenge by McDonald and Taylor (1959). We became aware that resonance is the missing phenomenon in hemodynamics (Lin Wang et al., 1991, 2004a). We analyzed from anatomic point of view that the major function of heart is not to emit waves associated with the axial blood motion but to provide pulsatile force to cause the aorta conducting a distributed radial oscillatory motion which can be represented and measured either by the distributed radius pulse of the artery $R(z, t)$ or the accompanied pressure pulse $P(z, t)$ (Lin Wang et al., 2007, Lin Wang and Wang, 2014a). To describe this radial motion, we derived a low dissipative pressure-radius (PR) wave equation for a fluid -filled elastic tube from Newton's law (Lin Wang et al., 2004b; Lin Wang and Wang, 2014b).

Arteries are blood-filled elastic vessels. Resonance phenomena for many different types of vibrations have been studied but the resonant behavior of a fluid-filled elastic tube has not drawn much attention yet. For the present study, trying to comprehend Frank's arterial resonance model, we conducted experiments to examine the pressure along a long elastic tube in response to either an impulsive or a periodic water input.

2. Methods

2.1. Components

The experimental setup to examine the resonance behavior of a long elastic tube is shown in Fig. 1.

2.1.1. Tubes

Latex tubes (Qualatex, Australia) with a Peterson's modulus $E_P = R(\partial P/\partial R)$ (Peterson et al., 1960) value of 0.6×10^5 Nm^{-2}, an inner diameter of 11.38 mm, wall

* Corresponding author.
E-mail address: yuhying@phy03.phy.ntnu.edu.tw (Y.-Y. Lin Wang).

http://dx.doi.org/10.1016/j.jbiomech.2015.02.026
0021-9290/

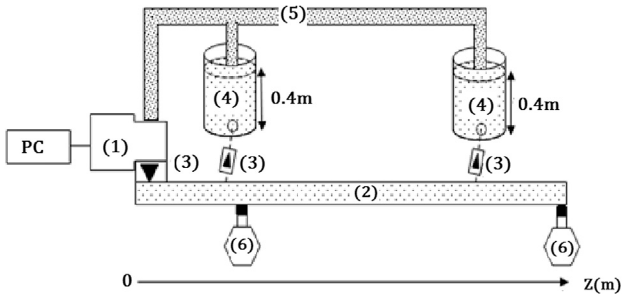

Fig. 1. Main components of the experimental setup: (1) near $z=0$ m, a PC controlled stepping motor, (2) a water-filled latex tube with length L (3) one-way valves, (4) water reservoirs, (5) silicon tubes, and (6) pressure transducers. The dashed lines indicate the level of water in the reservoirs.

thickness 0.31 mm and length either $L=1.80$ m or $L=1.20$ m were used (Fig. 1). Each elastic tube was fixed by laying down on a horizontal wooden groove with length 2.00 m. The two ends of the tube were occluded. In order to guide the flow of the water into the reservoirs, small holes were set at positions $z=L/6$ and $z=5\,L/6$ of the tube.

2.1.2. Pump

The ASM46AA stepping motor (Oriental Motor, USA) capable of generating either a periodic pulse or a single impulse signal was used to drive a pump. The stepping motor was controlled by a computer and an SFG-2004 function generator (GW Instek) which enabled us to adjust the duration for every water ejection and the ejection period. The pump had a one-way valve to prevent water from flowing backward and was connected to the elastic tube near one end of the tube, or at $z \cong 0$.

2.1.3. Reservoirs

Reservoirs were connected perpendicularly to the main elastic tube via one-way valves and long silicone tubes. The water level in the reservoirs was adjusted to be 0.40 m above the longitudinal axis of the main tube, thereby producing an initial hydrostatic pressure of 40 cm H_2O.

2.2. Instrumentation

The pressure responses were measured for 10 s using a pressure transducer (DP103, Validyne, USA) at various positions along the main tube. The full Scale of the transducer is 0.8 PSI or 56.2 cm-H_2O and the accuracy is 0.25% of the full Scale or 0.14 cm-H_2O. The pressure signals were amplified by a signal amplifier (CD223, Validyne) and then transferred to a PC via an A/D converter (PCI-9111, ADLINK, Taiwan) at a sampling rate of 4096 Hz.

2.3. Experimental procedures

Each measurement was repeated five times to enable statistical evaluations and to ensure the reliability and reproducibility of the data.

2.4. Measure the responding pressure induced by a single impulsive water ejection

We first sent a single impulse signal to let the stepping motor rotate one revolution in 50 ms, which drove the pump to inject impulsively 0.7 ml of water into the elastic tube. The impulse pressure responses at two axial locations z_1 and z_2 were recorded simultaneously by two pressure transducers. The two location sites z_1 and z_2 were chosen from $z=L/3$, $z=L/2$ and z near L. The pulse wave velocity $C=6.9\pm0.3$ m/s was determined by $|z_1-z_2|/\Delta t$ with Δt being the arrival time difference for the wave fronts of the two recorded pressures. Pressure responses measured in the time domain at various sites were transformed to the frequency domain by FFT; and the peaks in the frequency domain were identified as the damped natural frequencies (f_n^*) of the tube.

2.4.1. Measure the responding pressure induced by periodic water ejection

The stepper motor was set to rotate in a pulsatile fashion to inject 0.7 ml water periodically into the tube from the reservoir during each revolution. The PC controlled the water pumping frequency and the time duration of each ejection. The ratio of the duration time to the period of the water input, or ejection fraction (Reant et al., 2010), was chosen to be 30% to mimic left ventricular ejections (Weissler et al., 1963).

For various input periods, the pressure responses at different sites were measured in the time domain and transformed to the frequency domain by FFT.

To find the magnitude effect of the input, ejective volume of the water was then changed from 0.7 ml to 1.0 ml per pulse.

3. Results

3.1. Natural frequencies of the elastic tube

Fig. 2a shows the response pressure in the time domain initiated by a single impulsive water input near $z=1.80$ m for the elastic tube with length $L=1.80$ m; it manifest as a damped oscillation. Fig. 2b shows the response pressure in the frequency domain, and the peaks of the response occurred at 1.7 ± 0.1 Hz, 3.5 ± 0.1 Hz, 5.3 ± 0.1 Hz, which were identified as the damped resonant frequencies $f_1^*, f_2^*, f_3^*, \ldots$ of the elastic tube.

3.2. The natural frequencies are inversely proportional to the length L

The measured fundamental damped resonant frequencies (f_1^*) for the elastic tubes of length 1.80 m and 1.20 m, are 1.7 ± 0.1 Hz and 2.6 ± 0.1 Hz respectively. The experimental results shows that the natural frequencies of elastic tube systems are inversely proportional to their total length L.

3.3. The steady state pressure response for a periodic water input

Fig. 3 shows the steady state pressure response measured near the end of the elastic tube with length $L=1.80$ m as the pumping frequency f_P was fixed to 1.7 Hz which is the fundamental damped resonant frequency f_1^*.

3.4. Resonance phenomena

In response to any periodic water input, the pressure in the elastic tube reaches and maintains in a steady oscillation with the same period after a short transient time.

The pulse pressure is defined as the maximal change of pressure, or the highest pressure minus the lowest pressure in the periodic pressure response. The pulse pressure measured near $z=1.80$ m versus the pumping frequency f_P for the frequencies

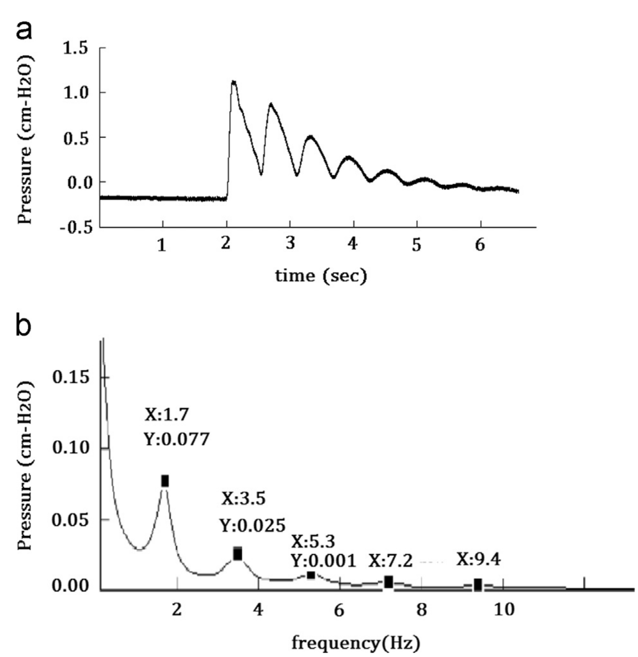

Fig. 2. The measured impulse response of the pressure near $z=1.80$ m of a latex tube with length 1.80 m in (a) the time domain and in (b) the frequency domain.

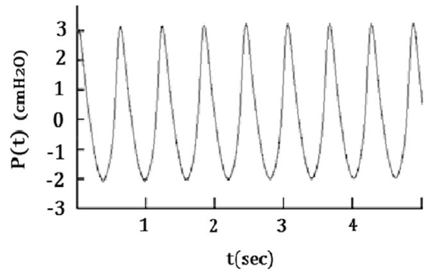

Fig. 3. Periodic pressure response of the 1.80 m latex tube measured near $z=1.80$ m, the pumping frequency of the stepping motor was fixed at 1.7 Hz.

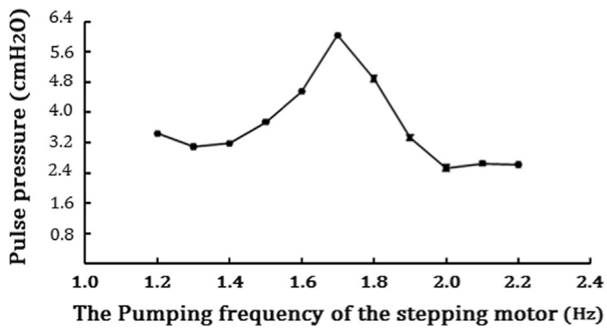

Fig. 4. Pulse pressure of the periodic pressure response of the 1.80 m latex tube measured near $z=1.80$ m, for pumping frequencies of the stepping motor between 1.2 and 2.2 Hz. (The SD is represented by the size of the marker).

varying from 1.2 to 2.2 Hz is shown in Fig. 4; the size of the marker represents the standard deviation of the five repeated experiments, and all the SD are within five percent of the pressure signals.

Fig. 4 shows that the peak of the responded pulse pressure occurred at $f_P = 1.7$ Hz for the elastic tube of length 1.80 m which is its fundamental damped resonant frequency f_1^*.

3.5. The distributed behavior of the pressure response

With the pumping frequency f_P being fixed to 1.7 Hz, amplitudes of the first three harmonic components of the periodic pressure response measured at $z=L/2$ and $z=L/3$ along the elastic tube of length $L=1.80$ m are shown in Fig. 5. At $z=L/2$, the amplitude of its second harmonic component is enriched as compared with its first harmonic component.

3.6. The effect of the injective volume on the pressure response

As we changed the injective volume of the water from 0.7 ml to 1.0 ml per pulse, the pulse pressure increased by approximately 25%.

4. Discussion

4.1. The water ejection provides a force to the elastic tube

In this study, the water ejected from the pump changes its momentum as it encounters the water-filled elastic tube (Fig. 1); based on Newton's law, it provides a force acting on the system and induces a distributed response pressure along the tube (Lin Wang and Wang, 2014a).

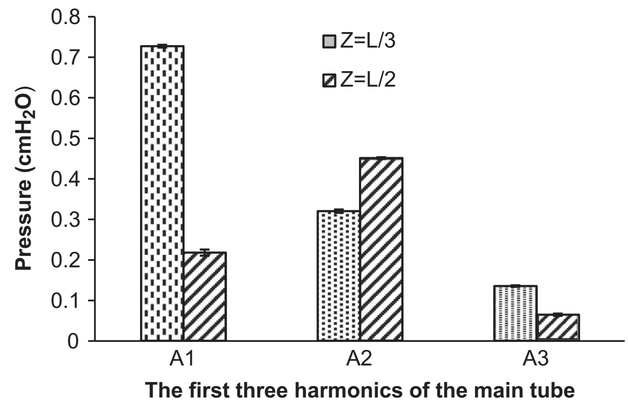

Fig. 5. The amplitudes of the first three harmonic components of the pressure measured at $z=L/3$ and $z=L/2$ along the main tube. The pumping frequency of the stepping motor was fixed at its fundamental damped resonant frequency (1.7 Hz).

4.2. The coupling of the elastic tube causes a filter effect to select out its natural frequencies

When the pump ejects a single impulsive water, it generates an impulsive force. Since the Furious integral transform (Kreyszig, 2006) of an impulsive function of short duration in the time domain is constant in the frequency domain, this force comprises harmonic waves of all frequencies (or in terms of the terminology of optics, a white wave). In spite of the white wave character of the input force, the whole elastic tube has a filter effect to select out the harmonic waves of frequencies $f_n{*} = \omega_n{*}/(2\pi)$ as can be observed from the peak locations of the pressure response in the frequency domain (Fig. 2b).

4.3. The induced pressure is governed by a low dissipation wave equation

If the wave mechanism in a tube system is of high dissipative character, a single impulsive water input would induce a single traveling response pressure that has significant space attenuation. The response pressure also would decay rapidly at each location so that it would not overlap with the response pressure initiated by the next impulsive water input. Since there is no accumulation effect by the repeated periodic water input, no resonance phenomena could be observed for a high dissipative system. However, if the wave mechanism is of low dissipative character, a single impulsive water input would induce a low attenuated traveling response pressure and at each location the response pressure would perform a low damped oscillation. The low damped oscillation is able to overlap with the new arriving response pressure initiated by the next impulsive water input. In this manner, the superposition of all the responses initiated by the repeated periodic input will reach a steady oscillatory state and manifests as a distributed stationary wave after a short transient time which makes the resonance behavior plausible.

Fig. 2a shows that for a single impulsive water input, the response pressure near the end of the tube does not just decay exponentially, but oscillates more than five times with the period corresponding to the natural frequency of the tube; it reveals the low damping character of the response pressure. Thus, the experimental results imply that the pressure wave equation in the water-filled elastic tube with a Peterson's modulus of 0.6×10^5 Nm^{-2} is governed by a low dissipated mechanism such as the PR wave equation which is associated mainly with the oscillatory motion of the tube radius $R(z, t)$ (Lin Wang and Wang, 2014b).

4.4. Resonance occurs when the frequency of the input is near the fundamental natural frequency of the system

A periodic water input induces a steady distributed pressure oscillation in the elastic tube, and Fig. 4 shows that that optimal pressure response or resonance occurs when the frequency of the input is near the fundamental natural frequency of the system.

4.5. The distributed arterial pressure could also be analyzed by the same method

Those analyzing methods are applicable for tube systems that are not stiff with the magnitude of the elastic modulus E_P being not much greater than the pressure P (Lin Wang et al., 2007); have length not much greater than the wavelength of the pressure (Migulin, 1983); and have the energy density associated with the axial flow Q to be much less than the pressure energy density P. In aorta, the energy density associated with Q is less than 5% of that associated with the pressure P (Milnor, 1989, Lin Wang and Wang, 2013); the mean blood pressure is about 100 mmHg or 1.3×10^4 newton/m^2 and the magnitude of E_P in a large artery has been reported to be of the order 5×10^4 newton/m^2 for humans (Milnor, 1989) which is not much greater than the pressure P; and the fundamental wavelength in artery can be estimated by PWV/heart rate or (5 m/sec)/(1/sec)=5 m, which is also of comparable order as the length of the aorta. Furthermore, the attenuation of the pressure wave along the aorta is low, hence the distributed forced radial oscillatory motion could also be a suitable approach to analyze the pressure wave in the aortic system.

4.6. Comparing with Frank's arterial resonance model

Frank (1899) and his followers lumped the whole arterial system together to a simple 0-D oscillator and got only one natural frequency for the system. But for a long stretched cylindrical tube performing a distributed-steady-periodic forced vibration in the radial direction, it has more than one natural frequency because the pressure $P(z,t)$ is a 1-D oscillatory function of the axial position z and the time t. The input force supplying by the ventricular blood input with period T_H can be decomposed by Fourier series analysis into many harmonic forces of discrete frequencies $f_n = n/T_H$, with $n = 1, 2, 3, \ldots$; hence having more than one natural frequency for the arterial system will make the resonance response to every harmonic input force possible to achieve the optimal power saving.

Conflict of interest

There is no any conflict of interest.

Acknowledgment

This study was supported by a grant from the National Science Council of R.O.C. (Grant no. NSC 100–2112-M-003 –002).

References

Frank, O., 1899. Die Grundform des arteriellen pulses: mathematische analyse. Z. Biol. 37, 483–526.
Kreyszig, E., 2006. Advanced Engineering Mathematics, 9th ed. Wiley & Sons, New York.
Li, J.K.-J., 2000. The Arterial Circulation: Physical Principles and Clinical Applications. Humana Press, Totowa, New Jersey.
Lin Wang, Y.Y., Chang, S.L., Wu, Y.E., Hsu, T.L., Wang, W.K., 1991. Resonance. The missing phenomenon in hemodynamics. Circ. Res. 69 (1), 246–249.
Lin Wang, Y.Y., Jan, M.Y., Shyu, C.S., Chiang, C.A., Wang, W.K., 2004a. The natural frequencies of the arterial system and their relation to the heart rate. IEEE Trans. Biom. Eng. 51 (1), 193–195.
Lin Wang, Y.Y., Jan, M.Y., Wang, G.C., Bau, J.G., Wang, W.K., 2004b. Pressure pulse velocity is related to the longitudinal elastic properties of the artery. Physiol. Meas. 25, 1397–1403.
Lin Wang, Y.Y., Chiu, W.B., Jan, M.Y., Bau, J.G., Li, S.P., Wang, W.K., 2007. Analysis of transverse wave as a propagation mode for the pressure pulse in large arteries. J. Appl. Phys. 102, 064702.
Lin Wang, Y.Y., Wang, W.K., 2013. A hemodynamics model to study the collective behavior of the ventricular-arterial system. J. Appl. Phys. 113 (2), 024702.
Lin Wang, Y.Y., Wang, W.K., 2014a. Anatomy of arterial systems reveals that the major function of the heart is not to emit waves associated with the axial blood motion. J. Physiol. 592 (2), 409.
Lin Wang, Y.Y., Wang, W.K., 2014b. PR wave equation - a primary and realistic arterial pressure wave equation for quantitative and collective study of the cardiovascular system. Chin. J. Phys. 52.2, 916–926.
McDonald, D.A., Taylor, M.G., 1959. The hydrodynamics of the arterial circulation. Prog. Biophys. Biophys. Chem. 9, 107–173.
Migulin, V.V. (Ed.), 1983. Basic Theory of Oscillations. Mir Publishers, Moscow.
Milnor, W.R., 1989. Hemodynamics, 2nd ed. Williams & Wilkins, Baltimore, MD.
Parker, K.H., 2009. A brief history of arterial wave mechanics. Med. Biol. Eng. Comput. 47 (2), 111–118.
Peterson, L.H., Jensen, R.E., Parnell, J., 1960. Mechanical properties of arteries in vivo. Circ Res 8, 622–639.
Reant, P., Dijos, M., Donal, E., Mignot, A., Ritter, P., Bordachar, P., Santos, P., Leclercq, C., Roudaut, R., Habib, G., Lafitte, S., 2010. Systolic time intervals as simple echocardiographic parameters of left ventricular systolic performance: correlation with ejection fraction and longitudinal two-dimensional strain. Eur. J. Echocardiogr., jeq084.
Remington, J.W., Hamilton, W.F., 1945. The construction of a theoretical cardiac ejection curve from the contour of the aortic pressure pulse. Am. J. Physiol. Content 144 (4), 546–556.
Remington, J.W., Hamilton, W.F., 1947. Quantitative calculation of the time course of cardiac ejection from the pressure pulse. Am. J. Physiol. 148 (1), 25–34.
Weissler, A.M., Harris, L.C., White, G.D., 1963. Left ventricular ejection time index in man. J Appl Physiol 18 (919–923), 375.